Cosmic ray threshold anomaly
and kinematics in the dS spacetime

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Abstract

We present a covariant framework of kinematics in the dS spacetime, which is a natural postulate of recent astronomical observations ($\Lambda > 0$). One-particle states are presented explicitly. It is noticed that the dispersion relation of free particles is dependent on the degrees of freedom of angular momentum and spin. This fact can be referred to as the effects of the cosmological constant on kinematics of particles. The kinematics in dS spacetime is used to investigate the phenomenon of ultra high energy cosmic rays. We emphasize the possibility of solving the threshold anomalies of the interactions between ultra high energy cosmic rays and soft photons in the covariant framework of kinematics in the dS spacetime.

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1 Introduction

The origin of the ultra-high energy cosmic rays (UHECR) is one of the outstanding puzzles of modern astrophysics. Today's understanding of the phenomena responsible for the production of UHECR is still limited. Currently, there are generally two categories of production mechanisms of the UHECR. One is the "bottom-up" acceleration scenario with some astrophysical objects as sources[1, 2]. The other is called "top-down" scenario in which UHECR particles are from the decay of certain sufficiently massive particles originating in the early Universe[3].

Decades ago, Greisen, Zatsepin and Kuzmin[4] discussed the propagation of the UHECR particles through the cosmic microwave background radiation (CMBR). Due to the photopion production process by the CMBR, the UHECR particles will lose their energies drastically down to a theoretical threshold, which is about 5×10^{19}eV. That is to say, the mean free path for this process is only a few Mpc[5]. This is the so-called GZK cutoff. However, we have observed indeed hundreds of events with energies above 10^{19}eV and about 20 events above 10^{20}eV[6]-[10]. At the same time, there is another paradox[11] in the terrain of cosmic ray, which comes from the detected 20TeV photons from the MrK 501 (a BL Lac object at a distance of 150Mpc). Similar to the case of UHECR, due to pair production process by the IR background photons, the 20TeV photons should have disappeared before arrival at the ground-based detectors. Both of the puzzles can be considered to be some cosmic ray threshold anomalies: energy of an expected threshold is reached but the threshold has not been observed yet.

Recently, there is growing interest on the theory of doubly special relativity (DSR)[12, 13]. In the DSR, there exist two observer-independent scales. One of them is a scale of velocity, which is identified with the velocity of light. The other is a scale of mass/length, which is expected to be the order of the Planck mass/length. In fact, the violation of the Lorentz invariance and the Planck scale physics have long been studied as possible solutions of the cosmic ray threshold anomalies[14]-[20]. However, all of these scenarios are far beyond the standard cosmological theory and the standard model of particle physics. Furthermore, it is well-known that we are still very far from

a theory of quantum gravity, in spite of extensive investigations on candidates such as the supergravity, Kluza-Klein, noncommutative geometry and superstring theory.

The recent astronomical observations on supernovae [21, 22] and CMBR [23] show that about two thirds of the whole energy in the Universe is contributed by a small positive cosmological constant (Λ). An asymptotic de Sitter (dS) spacetime is premised naturally. The physics in an asymptotic dS spacetime has been discussed extensively[24]-[26].

In this paper, we discuss kinematics in an asymptotic dS spacetime. The framework of classical as well as quantum kinematics in the dS spacetime is set up carefully. We get a general form of dispersion relation for free particles in the dS spacetime. This formalism is used to describe the UHECR propagating in the cosmic microwave background as well as the TeV-γ propagating in the infrared background. We obtain explicitly the corrections of the GZK threshold for the UHECR particles interacting with soft photons, which are dependent on the cosmological constant as supposed in the beginning of the paper. We show how the threshold varies with a positive cosmological constant and additional degrees of freedom of the angular momentums of interacting particles. It should be noticed that, for a positive cosmological constant, the theoretic threshold tends to be above the energies of all the observed events. Thus, we may conclude that the tiny but nonzero cosmological constant is a possible origin of the threshold anomalies of the UHECR and the TeV-γ.

The paper is organized as follows. In Section 2, we discuss the classical kinematics in the dS spacetime. Conservation laws of momentum and angular momentum are obtained along the geodesics. Section 3 is devoted to the investigation of the quantum kinematics in the dS spacetime. By solving equations of motion of a free particle, we present a remarkable dispersion relation for free particles in the dS spacetime, which includes degrees of freedom of angular momentum and spin. In Section 4, by taking effects of a tiny but nonzero positive cosmological constant into account, we show that the theoretic threshold is above the energies of all the observed UHECR events. Similar discussion is made for the TeV-γ in Section 5. In the end, we present conclusions and

remarks.

2   Classical kinematics

The dS spacetime can be realized as a four dimensional pseudo-sphere imbedded in the five dimensional flat space

$$\left(\xi^0\right)^2 - (\xi^1)^2 - (\xi^2)^2 - (\xi^3)^2 - \left(\xi^5\right)^2 = -\frac{1}{\lambda} \ ,$$

$$ds^2 = (d\xi^0)^2 - (d\xi^1)^2 - (d\xi^2)^2 - (d\xi^3)^2 - (d\xi^5)^2 \ , \tag{1}$$

where λ is the Riemannian curvature of the dS spacetime. It is obvious that the above equations are invariant under the action of the de Sitter group $SO(1, 4)$. The coordinates ξ^μ ($\mu = 0, 1, 2, 3, 5$) are related to the Beltrami coordinates x^i ($i = 0, 1, 2, 3$) through the relation

$$x^i = \frac{\xi^i}{\sqrt{\lambda}\xi^5} \ , \qquad (\xi^5 \neq 0) \ . \tag{2}$$

Then, in the Beltrami coordinate, we can write the dS spacetime as[31, 32]

$$\sigma \equiv \sigma(x, \ x) = 1 - \lambda\eta_{ij}x^i x^j > 0 \ ,$$

$$ds^2 = \left(\frac{\eta_{ij}}{\sigma} + \frac{\lambda\eta_{ir}\eta_{js}x^r x^s}{\sigma^2}\right) dx^i dx^j \ , \tag{3}$$

where $\eta_{ij} = \text{diag}(1, \ -1, \ -1, \ -1)$ is the Minkowski metric. Transformations of x^i acted by $SO(1, 4)$ can be expressed as follows,

$$x^i \to \tilde{x}^i = \sigma(b, \ b)^{\frac{1}{2}}\sigma(b, \ x)^{-1}(x^j - b^j)D^i_j \ ,$$

$$D^i_j = L^i_j + \lambda \left(\sigma(b, \ b) + \sigma(b, \ b)^{\frac{1}{2}}\right)^{-1} \eta_{kl}b^l b^i L^k_j \ ,$$

$$L \equiv (L^i_j) \ \in SO(3, \ 1) \ , \tag{4}$$

$$\sigma(b, \ b) > 0 \ ,$$

where (b^i) is an arbitrary point in the Beltrami dS spacetime. We can define the five dimensional angular momentum $M^{\mu\nu}$ of a free particle with mass m_0 as the form

$$M^{\mu\nu} = m_0 \left(\xi^\mu \frac{d\xi^\nu}{ds} - \xi^\nu \frac{d\xi^\mu}{ds}\right) \ , \tag{5}$$

where s is a parameter along the geodesic. In the dS spacetime, there is no translation invariance and so that one can not introduce a momentum vector. However, it should be noticed that, at least somehow, we may define a counterpart of the momentum P for a free particle in the dS spacetime

$$P^i \equiv \sqrt{\lambda} M^{5i} = m\sigma^{-1} \frac{dx^i}{ds} , \qquad (i = 0,\ 1,\ 2,\ 3) . \tag{6}$$

In the same manner, the counterparts of the four dimensional angular momentum J^{ij} can be assigned as

$$J^{ij} \equiv M^{ij} = x^i P^j - x^j P^i = m\sigma^{-1} \left(x^i \frac{dx^j}{ds} - x^j \frac{dx^i}{ds} \right) . \tag{7}$$

It is not difficult to show that, along the geodesics, the above defined momentum and angular momentum are invariant,

$$\frac{dP^i}{ds} = 0 , \qquad \frac{dJ^{ij}}{ds} = 0 . \tag{8}$$

In fact, these are the equations of geodesic in the dS spacetime. From Eq. (8), we get, for a free particle,

$$\frac{dx^\alpha}{dx^0} = \text{constant} , \qquad (\alpha = 1,\ 2,\ 3) . \tag{9}$$

The fact tells us that the Beltrami coordinate can be considered to be an inertial frame[33] as the Cartesian coordinate in the Minkowski spacetime. We notice that there is a subgroup $SO(4)$ of the de Sitter one $SO(1,4)$, which consists of spatial transformations among x^α. It is easy to show that $\xi^0 (\equiv \sigma(x,x)^{-1/2} x^0)$ is invariant under the spatial transformations. Thus, we can say two spacelike events are simultaneous if they satisfy

$$\sigma(x,\ x)^{-\frac{1}{2}} x^0 = \xi^0 = \text{constant} . \tag{10}$$

Therefore, it is convenient to discuss physics of the dS spacetime in the coordinate $(\xi^0,\ x^\alpha)$. In this coordinate, the metric can be rewritten into the form

$$ds^2 = \frac{d\xi^0 d\xi^0}{1 + \lambda \xi^0 \xi^0} - (1 + \lambda \xi^0 \xi^0) \left[\frac{d\rho^2}{(1 + \lambda \rho^2)^2} + \frac{\rho^2}{1 + \lambda \rho^2} d\Omega^2 \right] , \tag{11}$$

5

where $\rho^2 \equiv \Sigma x^\alpha x^\alpha$ and $d\Omega^2$ denotes the metric on 2-dimensional sphere S^2.

If a proper time τ is introduced as

$$\tau \equiv \frac{1}{\sqrt{\lambda}} \sinh^{-1}(\sqrt{\lambda}\xi^0) , \tag{12}$$

one would get a Friedman-Robertson-Walker like metric,

$$ds^2 = d\tau^2 - \cosh^2(\sqrt{\lambda}\tau) \left[\frac{d\rho^2}{(1 + \lambda\rho^2)^2} + \frac{\rho^2}{1 + \lambda\rho^2} d\Omega^2 \right] . \tag{13}$$

In terms of the five dimensional angular momentum $M^{\mu\nu}$, we can construct an invariant under the de Sitter transformations for a free particle[34],

$$m_0^2 = \frac{\lambda}{2} M^{\mu\nu} M_{\mu\nu} = E^2 - \mathbf{P}^2 + \frac{\lambda}{2} J^{ij} J_{ij} , \tag{14}$$

$$E = P^0 , \quad \mathbf{P} = (P^1, \ P^2, \ P^3) .$$

However it should be noticed that the P^i defined in this way is not a four dimensional vector but the component of the five dimensional angular momentum $M^{\mu\nu}$.

3   Quantum kinematics

It is natural to realize the five dimensional angular momentum $M_{\mu\nu}$ as infinitesimal generators of the de Sitter group $SO(1, \ 4)$

$$M^{\mu\nu} = -i \left(\xi^\mu \frac{\partial}{\partial \xi_\nu} - \xi^\nu \frac{\partial}{\partial \xi_\mu} \right) . \tag{15}$$

The de Sitter invariant, or the Casimir operator can be used to express the one-particle states in the dS spacetime

$$\left(\frac{\lambda}{2} M^{\mu\nu} M_{\mu\nu} - m_0^2 \right) \Phi(\xi^0, x^\alpha) = 0 , \tag{16}$$

where the $\Phi(\xi^0, x^\alpha)$ denotes a scalar field or a component of vector fields with a given spin s.

In the coordinates $(\xi^0, \ x^\alpha)$, we can rewrite the Casimir operator as the following form

$$
\begin{aligned}
\frac{\lambda}{2} M^{\mu\nu} M_{\mu\nu} =\ & -\left(1 + \lambda\xi^0\xi^0\right) \partial_{\xi^0}^2 - 4\lambda\xi^0 \partial_{\xi^0} \\
& + \left(1 + \lambda\xi^0\xi^0\right)^{-1} \left(1 + \lambda\rho^2\right)^2 \left[\partial_\rho^2 + 2\rho^{-1}\partial_\rho \right] \\
& + \left(1 + \lambda\xi^0\xi^0\right)^{-1} \left(1 + \lambda\rho^2\right) \rho^{-2} \left(\partial_{\mathbf{u}}^2 - s(s+1) \right) ,
\end{aligned} \tag{17}
$$

where $\mathbf{u}\mathbf{u}' = 1$ and $\partial_{\mathbf{u}}^2$ denotes the Laplace operator on S^2.

To solve the equation of motion, we write the field $\Phi(\xi^0, x^\alpha)$ into the form

$$\Phi(\xi^0, \rho, \mathbf{u}) = T(\xi^0)U(\rho)Y_{lm}(\mathbf{u}) \ .$$

Thus, we transform the equation of motion into [26, 34],

$$\left[(1 + \lambda\xi^0\xi^0)^2\partial_{\xi^0}^2 + 4\lambda\xi^0(1 + \lambda\xi^0\xi^0)\partial_{\xi^0} + m_0^2(1 + \lambda\xi^0\xi^0) + (\varepsilon^2 - m_0^2)\right] T(\xi^0) = 0,$$

$$\left[\partial_\rho^2 + \frac{2}{\rho}\partial_\rho - \left[\frac{m_0^2 - \varepsilon^2}{(1 + \lambda\rho^2)^2} + \frac{l(l+1) + s(s+1)}{\rho^2(1 + \lambda\rho^2)}\right]\right] U(\rho) = 0,$$

$$\left[\partial_{\mathbf{u}}^2 + l(l+1)\right] Y_{lm}(\mathbf{u}) = 0, \tag{18}$$

where $Y_{lm}(\mathbf{u})$ is the spherical harmonic function and ε is a constant.

Solutions of timelike part of the field are of the forms

$$T(\xi^0) \sim \left(1 + \lambda\xi^0\xi^0\right)^{-1/2} \cdot \begin{cases} P_\nu^\mu(i\sqrt{\lambda}\xi^0) \ , \\ \\ Q_\nu^\mu(i\sqrt{\lambda}\xi^0) \ , \end{cases} \tag{19}$$

where μ, ν satisfy

$$\begin{aligned} \nu(\nu + 1) &= 2 - \lambda^{-1}m_0^2 \ , \\ \mu^2 &= 1 + \lambda^{-1}(\varepsilon^2 - m_0^2) \ . \end{aligned} \tag{20}$$

For the radial equation of the field, we can write the solutions as the form

$$U(\rho) \sim \rho^l(1 + \lambda\rho^2)^{k/2}F\left(\frac{1}{2}(l + k + s + 1), \ \frac{1}{2}(l + k + s), \ l + s + \frac{3}{2}; \ -\lambda\rho^2\right) , \tag{21}$$

where k denotes the radial quantum number

$$k^2 - 2k - \lambda^{-1}(\varepsilon^2 - m_0^2) = 0 \ .$$

To be normalizable, the hypergeometric function in the radial part of the wavefunction has to break off, leading to the quantum condition

$$\frac{l + k + s}{2} = -n \ , \qquad (n \in \mathbf{N}) \ . \tag{22}$$

Then, we obtain the dispersion relation for a free particle

$$E^2 = m_0^2 + \varepsilon'^2 + \lambda(2n + l + s)(2n + l + s + 2) \ . \tag{23}$$

4   UHECR threshold anomaly

Until today, we have observed[6]-[10] hundreds of events with energies above 10^{19}eV and about 20 events above 10^{20}eV, which are above the GZK threshold. In principle, photopion production with the cosmic microwave background radiation photons should decrease the energies of these protons to the level below the corresponding threshold. In this section, we discuss the UHECR threshold anomaly in the covariant framework of kinematics in dS spacetime set up in the preceding sections.

We consider the head-on collision between a soft photon of energy ϵ, momentum $\mathbf{q}$ and a high energy particle m_1 of energy E_1, momentum $\mathbf{p}_1$, which leads to the production of two particles m_2, m_3 with energies E_2, E_3 and momentums $\mathbf{p}_2$, $\mathbf{p}_3$, respectively. From the energy and momentum conservation laws, we have

$$E_1 + \epsilon = E_2 + E_3 \ ,$$
$$p_1 - q = p_2 + p_3 \ . \tag{24}$$

In the C.M. frame, m_2 and m_3 are at rest at threshold, so they have the same velocity in the lab frame and there exists the following relation

$$\frac{p_2}{p_3} = \frac{m_2}{m_3} \ . \tag{25}$$

It is convenient to use the approximated formulae of dispersion relations (23) for the soft photons and the ultra high energy particles

$$\epsilon^2 = q^2 + \lambda_\gamma^* \ , \tag{26}$$

$$E_i = \sqrt{m_i^2 + p_i^2 + \lambda_i^*} \simeq p_i + \frac{m_i^2}{2p_i} + \frac{\lambda_i^*}{2p_i} \ , \quad (i = 1, \ 2, \ 3) \ . \tag{27}$$

where $\lambda_\gamma^* \equiv \lambda(l_\gamma + 2n_\gamma + 1)(l_\gamma + 2n_\gamma + 3) \approx \lambda l_\gamma(l_\gamma + 1)$ and $\lambda_i^* \equiv \lambda(l_i + 2n_i + s_i)(l_i + 2n_i + s_i + 2) \approx \lambda l_i(l_i + 1)$ with the conjecture that $l \gg n, \ s$.

The obtained threshold can be expressed as the form

$$E_{\text{th},\,\lambda} \simeq \frac{(m_2 + m_3)^2 - m_1^2 + \lambda_2^* \left(1 + \frac{m_3}{m_2}\right) + \lambda_3^* \left(1 + \frac{m_2}{m_3}\right) - \lambda_1^*}{2\left(\epsilon + \sqrt{\epsilon^2 - \lambda_\gamma^*}\right)} \; . \tag{28}$$

The usual GZK threshold could be recovered when the parameter λ^*, which is dependent on the cosmological constant, runs to zero.

The conservation law of the angular momentum gives a constraint on the parameters λ^*,

$$\lambda_1^* + \lambda_\gamma^* + 2\lambda \mathbf{L}_1 \cdot \mathbf{L}_\gamma = \lambda_2^* + \lambda_3^* + 2\lambda \mathbf{L}_2 \cdot \mathbf{L}_3 \; .$$

Making use of the relation, we can rewrite the λ^* dependent terms of the threshold as the following

$$\frac{\lambda_2^* \frac{m_3}{m_2} + \lambda_3^* \frac{m_2}{m_3} + \lambda_\gamma^* + 2\lambda \mathbf{L}_1 \cdot \mathbf{L}_\gamma - 2\lambda \mathbf{L}_2 \cdot \mathbf{L}_3}{2\left(\epsilon + \sqrt{\epsilon^2 - \lambda_\gamma^*}\right)} \; . \tag{29}$$

If λ_2^* and λ_3^* take value of the same order with λ_γ^* (less than the square of energy of a soft photon), the λ^* dependent terms can be omitted[37]. We will investigate the case of $\lambda_2^* + \lambda_3^* \gg \lambda_\gamma^*$, and the threshold (28) is of the form

$$E_{\text{th},\,\lambda} \simeq \frac{(m_2 + m_3)^2 - m_1^2 + \lambda_2^* \frac{m_3}{m_2} + \lambda_3^* \frac{m_2}{m_3} - 2\lambda \mathbf{L}_2 \cdot \mathbf{L}_3}{2\left(\epsilon + \sqrt{\epsilon^2 - \lambda_\gamma^*}\right)} \; . \tag{30}$$

Now, we can study the photopion production processes of the UHECR interaction with the CMBR

$$p + \gamma \rightarrow p + \pi \; .$$

The corresponding threshold for this process is given by

$$E_{\text{th},\,\lambda}^{\text{UHECR}} \simeq \frac{(m_N + m_\pi)^2 - m_N^2 + \lambda_N^* \frac{m_\pi}{m_N} + \lambda_\pi^* \frac{m_N}{m_\pi} - 2\lambda \mathbf{L}_N \cdot \mathbf{L}_\pi}{2\left(\epsilon + \sqrt{\epsilon^2 - \lambda_\gamma^*}\right)} \; . \tag{31}$$

To show the behavior of the threshold in the λ^*-parameter space clearly, we should discuss some limit cases in detail.

In the case that the out-going nucleon has zero angular momentum, the threshold (31) reduces as

$$E_{\text{th, }\lambda,\pi}^{\text{UHECR}} \simeq \frac{(m_N + m_\pi)^2 - m_N^2 + \lambda_\pi^* \frac{m_N}{m_\pi}}{2\left(\epsilon + \sqrt{\epsilon^2 - \lambda_\gamma^*}\right)} \ . \tag{32}$$

We give a plot for the dependence of the threshold $E_{\text{th, }\lambda,\pi}^{\text{UHECR}}$ on the cosmological constant and angular momentum (the in-going photon and out-going pion) in **FIG.1**.
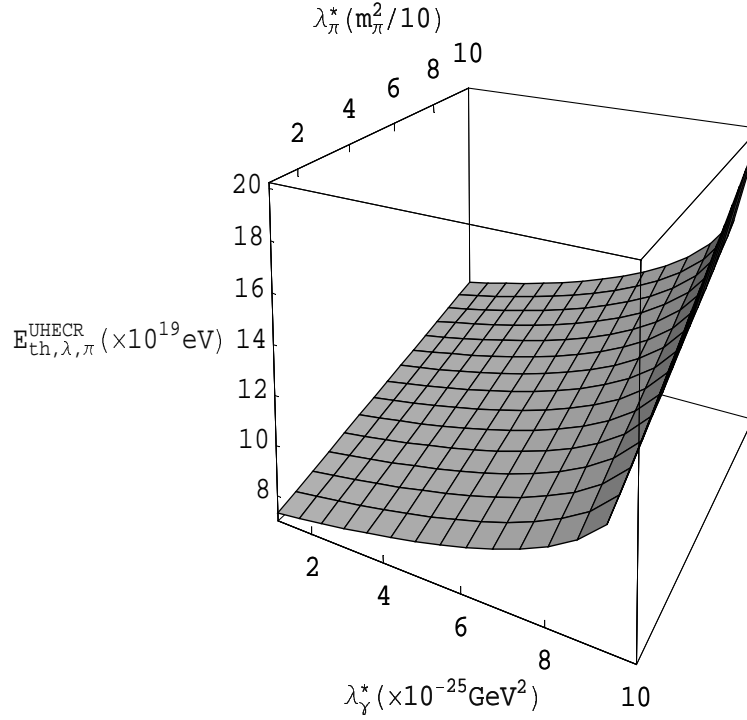

FIG.1   The cosmological constant and angular momentum (of in-going soft photon and out-going pion) dependence

of the threshold $E_{\text{th, }\lambda,\pi}^{\text{UHECR}}$ in the interaction between the UHECR protons and the CMBR photons (λ_π^* in unit of $m_\pi^2/10$) .

In the case that the out-going pion has zero angular momentum, the UHECR threshold takes the form

$$E_{\text{th, }\lambda,N}^{\text{UHECR}} \simeq \frac{(m_N + m_\pi)^2 - m_N^2 + \lambda_N^* \frac{m_\pi}{m_N}}{2\left(\epsilon + \sqrt{\epsilon^2 - \lambda_\gamma^*}\right)} \ . \tag{33}$$

We give a plot for the dependence of the threshold $E_{\text{th, }\lambda,N}^{\text{UHECR}}$ on the cosmological constant and angular momentum (the in-going soft photon and out-going nucleon) in **FIG.2**.
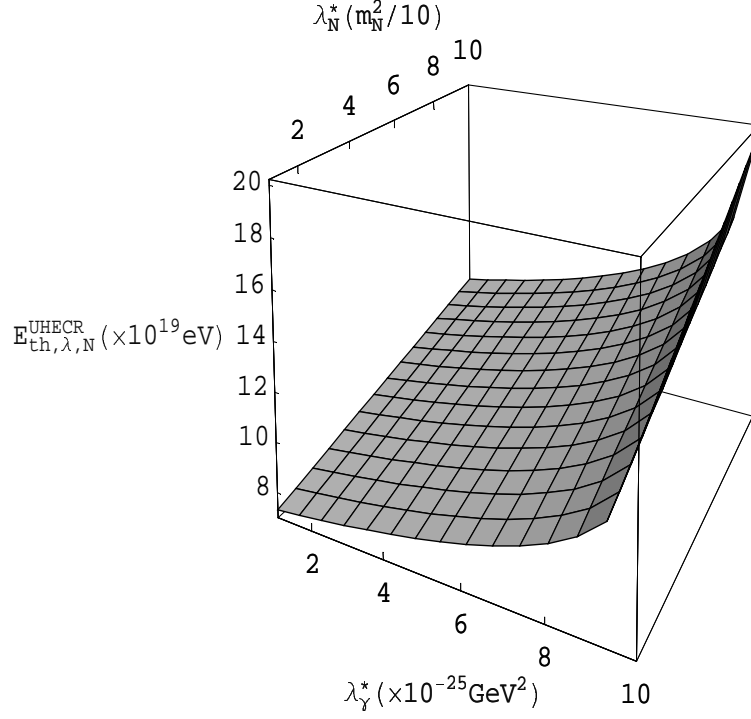
FIG.2 The cosmological constant and angular momentum (of the in-going soft photon and out-going nucleon) dependence of the threshold $E_{\text{th}, \lambda, N}^{\text{UHECR}}$ in the interaction between the UHECR protons and the CMBR photons (λ_N^* in unit of $m_N^2/10$) .

Finally, if the out-going pion and nucleon has the same angular momentum, the UHECR threshold can be expressed as the following form

$$E_{\text{th},\lambda,N\pi}^{\text{UHECR}} \simeq \frac{(m_N + m_\pi)^2 - m_N^2 + \lambda_N^*(\frac{m_N}{m_\pi} + \frac{m_\pi}{m_N} - 2)}{2\left(\epsilon + \sqrt{\epsilon^2 - \lambda_\gamma^*}\right)} \ . \tag{34}$$

We give a plot for the dependence of the threshold $E_{\text{th}, \lambda, N\pi}^{\text{UHECR}}$ on the cosmological constant and angular momentum (the out-going nucleon and pion) in **FIG.3**.

From the above discussion, we know that a tiny but nonzero cosmological constant may provide indeed sufficient corrections to the primary predicted threshold[4]. For the observed cosmological constant (which is around the level of 10^{-85}GeV^2), if the CMBR possesses a quantum number l_γ of the order of 10^{30}, the threshold will be above the energies of all those observed UHECR particles. The predicted threshold should be upgraded to a more reasonable level. Now we can say that a possible origin of the cosmic ray threshold anomaly has been got. It is the cosmological constant that

11

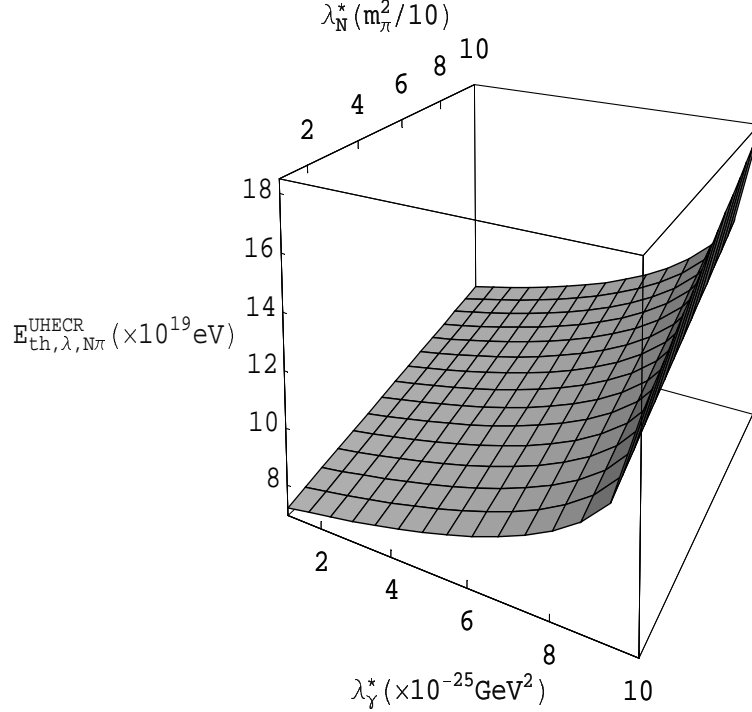increases the GZK cut-off to a level above the observed UHECR events.

FIG.3   The cosmological constant and angular momentum (of the out-going nucleon and pion) dependence of the threshold $E_{\mathrm{th},\,\lambda,N\pi}^{\mathrm{UHECR}}$ in the interaction between the UHECR protons and the CMBR photons ($\lambda_N^* = \lambda_\pi^*$ in unit of $m_\pi^2/10$) .

5   TeV-γ threshold anomaly

The second paradox for the cosmic ray comes from the fact that experiments detected 20TeV photons from Mrk 501 (a BL Lac object at a distance of 150Mpc). Similar to the UHECR case, due to the interaction with the IR background photons, the 20TeV photons should have disappeared before arrival at the ground-based detections. Parallel to the analysis of UHECR in the previous section, we discuss the TeV-γ threshold anomaly in the following.

For the pair production process

$$\gamma + \gamma \rightarrow e^+ + e^- \ ,$$

we obtain a threshold by taking m_1 in the equation (30) to be zero,

$$E^{\gamma}_{\text{th},\,\lambda} \simeq \frac{4m_e^2 + \lambda^*_{e^+} + \lambda^*_{e^-} - 2\lambda \mathbf{L}_{e^+} \cdot \mathbf{L}_{e^-}}{2(\epsilon + \sqrt{\epsilon^2 - \lambda^*_\gamma})} \ . \tag{35}$$

If one of the out-going particles possesses zero angular momentum, the corresponding threshold is reduced to the form

$$E^{\gamma}_{\text{th},\lambda,e} \simeq \frac{2m_e^2 + \lambda^*_e/2}{\epsilon + \sqrt{\epsilon^2 - \lambda^*_\gamma}} \ . \tag{36}$$

We present a plot of the dependence of the threshold $E^{\gamma}_{\text{th},\lambda,e}$ on the cosmological constant and angular momentum (the out-going electron or positron) in **FIG.4**.
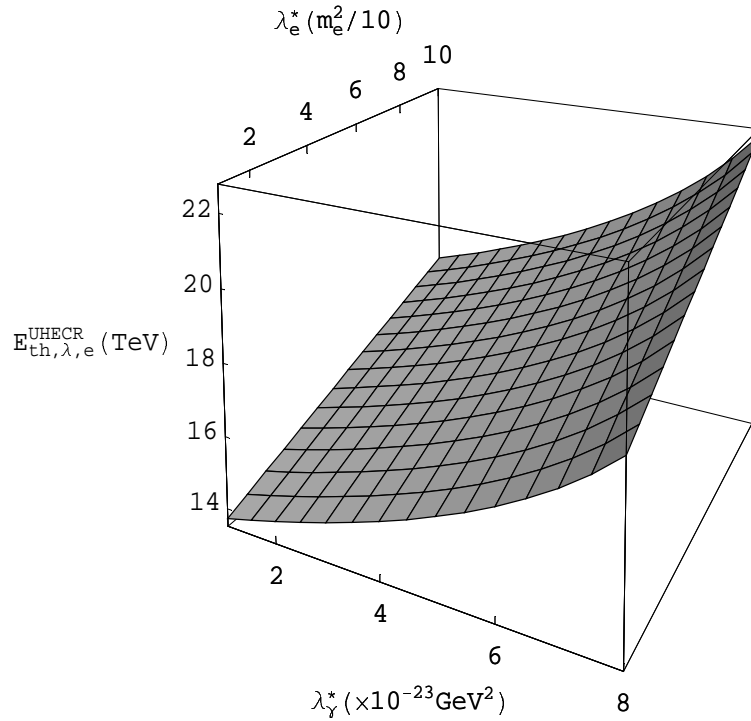

FIG.4   The cosmological constant and angular momentum (of the in-going soft photon and out-going electron) dependence of the threshold $E^{\gamma}_{\text{th},\lambda,e}$ in the interaction between the TeV-γ and the IR background photons (λ^*_e in unit of $m_e^2/10$) .

From the above plot, we see that the threshold $E^{\gamma}_{\text{th},\lambda}$ is also quite sensitive to the small positive cosmological constant. By taking into account the corrections which are dependent on the cosmological constant and angular momentum of interacting particles, we get a new predicted threshold. And then the threshold anomaly disappears.

6 Conclusions and remarks

In this paper, we discussed kinematics in the dS spacetime. The kinematic invariance group in the dS spacetime is $SO(1,4)$ instead of the Poincaré one in the Minkowski sapcetime. Kinematics, which is based on the de Sitter group $SO(1,4)$, was set up formally. It should be noticed that the dispersion relation for a free particle in the dS spacetime is related with degrees of freedom of angular momentum and spin. With the help of this deformed dispersion relation, and thanks to the positive cosmological constant, a possible origin of the threshold anomalies was proposed naturally.

We would like to point out that, if the cosmological constant can vary notably in different period of the Universe (at least in some scenarios it is), we may get a higher threshold for the UHECR propagating in the early Universe. And this could be a good news for the "top-down" scenario of the UHECR.

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